(b) Using Cauchy's test, discuss the convergence of the series : **2**

$$\sum_{n=3}^{\infty} \frac{1}{n \log n \left(\log (\log n)\right)}$$

Section IV

8. (a) Test the convergence and absolutely convergence of the series : **3**

$$\frac{1}{2(\log 2)^{P}} - \frac{1}{3(\log 3)^{P}} + \frac{1}{4(\log 4)^{P}} - \ldots\ldots\ldots (P > 0)$$

(b) Show that the series : **2**

$$\frac{\log 2}{2^2} - \frac{\log 3}{3^2} + \frac{\log 4}{4^2} - \ldots\ldots$$

converges.

9. (a) Show that the infinite product : **3**

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\ldots\ldots$$

is convergent.

(b) Discuss the convergence of cauchy product of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p} (p > 0)$ with itself. **2**

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No. of Printed Pages : 04 **Roll No. ....**

32209

B. A. & Hons. (Subsidiary)

EXAMINATION, 2025

(Fourth Semester)

(Re-appear Only)

MATHEMATICS

BM-241

Sequence and Series

Time : 3 Hours] [*Maximum Marks :* 27

Before answering the question-paper, candidates must ensure that they have been supplied with correct and complete question-paper. No complaint, in this regard will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

1. (a) Define limit point of a set. **1**

(b) Find infimum and supremum of the following set : **2**

$$\{2\ 2^2\ 2^3 \ldots\ldots\ldots 2^n\}$$

(c) Prove that Cauchy sequences bounded. **2**

(d) Define closure of a set with example. **1**

(e) State Bolzano-Weierstrass Theorem. **1**

Section I

2. (a) Prove that every non-empty set of real numbers which is bounded below has g.l.b. **3**

(b) Prove that set of rationals is not order complete. **2**

3. (a) Prove that every infinite bounded subset of real numbers has limit point. **3**

(b) Prove that '0' is the only limit point of set $\left\{\frac{1}{n} \middle| n \in \mathbb{N}\right\}$. **2**

Section II

4. (a) State and prove Cauchy first theorem on limits. **3**

(b) Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ exists and lies between 2 and 3. **2**

5. (a) A sequence is convergent iff it is cauchy. Prove it. **3**

(b) By Cauchy general principle of convergence prove that $a_n = 1+\frac{1}{3}+\frac{1}{5}+\ldots\ldots+\frac{1}{2^{n-1}}$ does not converge. **2**

Section III

6. (a) State and prove Raabe's test for convergence and divergence of infinite series. **3**

(b) Show that : **2**

$$\lim_{x\to\infty}\left(\frac{2}{1},\frac{3}{2},\frac{4}{3},\ldots\ldots\ldots,\frac{n}{n-1}\right)^{1/n} = 1$$

7. (a) Prove that monotonically increasing sequence $<a_n>$ which is bounded above converges to its least upper bound. **3**